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Selberg Trace Formula for odd weight and $\Gamma\not\ni -1$

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§0 Introduction

The main purpose of this note is to develop the Selberg Trace Formula for odd weight and $\Gamma\not\ni -1$. It is well known that there is a slight difference between the contribution of regular cusps and the one of irregular cusps even in the case of the dimension formula of the space of cusp forms of the above type . Anyone who wants to know this phenomenon closely, will find the lack of the Trace Formula of the above type.

The second purpose of this note is as follows. At the symposium "Automorphic Forms and Related Topics" in Research Institute, Kyoto Univ. Jan. 1987 , Tanigawa, Hiramatsu and the auther made a report about the Trace Formula for Hecke Operaters in the case of weight one. In the special case of the report, we wrote the dimension of the space of cusp forms of weight one, using the residue of the Selberg type zeta function. [5] The result is the extension of [2][6][7]. But the formula is unsatisfactory because the "zeta" has no functional equation. In this note, we write the dimension formula of weight one by more natural "zeta" function which *has* functional equation.

§1 Notation

Let $\mathbb{H}$ be the complex upper half plane and $T=\mathbb{Z}/(2\pi)$. We set $\tilde{\mathbb{H}}=\mathbb{H}\times T$, $G=SL(2,\mathbb{R})$, $\tilde{G}=G\times T$. Then $\tilde{G}$ acts transitively on $\tilde{\mathbb{H}}$ in the following way;

$$(g,\ \alpha)\cdot(z,\ \phi)=(\ g\cdot z\ ,\ \phi+\ \arg j(g,z)-\alpha)\qquad (g,\ \alpha)\in\widetilde{G}\ ,\ (z,\ \phi)\in\widehat{\mathbb{H}}$$

where $g=\begin{bmatrix} a & b \\ c & d \end{bmatrix}\in G$, $g\cdot z=\dfrac{az+b}{cz+d}\in\mathbb{H}$, $j(g,z)=cz+d$.

With the involution $\xi(z,\phi)=(-\bar{z},-\phi)$, $(G,\mathbb{H},\xi)$ is the Weakly Symmetric Riemannian Space. The ring of G invariant differential operaters on this space is generated by Δ and $\dfrac{\partial}{\partial\phi}$ where ,

$$\Delta=y^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+y\frac{\partial}{\partial x}\frac{\partial}{\partial\phi}\quad .$$

Let Γ be the discrete subgroup of G not containing -1.

We identify G with $G\times\{0\}$, and Γ with $\Gamma\times\{0\}$.

Take a unitary representation χ of Γ of degree ν ($<\infty$) . Let $\kappa_1,\kappa_2,\cdots,\kappa_\omega$ be the complete representatives of Γ inequivalent cusps of $\Gamma\backslash\mathbb{H}$. And we write Γ_i the stabilizer of κ_i, and $\Gamma_i^0=\Gamma_i\cap\ker\chi$. We will consider the cusp form of $\Gamma\backslash\mathbb{H}$, so take χ under the condition $[\Gamma_i,\Gamma_i^0]<\infty$ for $i=1,2,\cdots,\omega$.

Take $\sigma_i\in G$ such that $\sigma_i\infty=\kappa_i$, satisfying the following condition;

If κ_i is regular then $\Gamma_\infty=\sigma_i^{-1}\Gamma_i\sigma_i$ is generated by $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

If κ_i is irregular then $\Gamma_\infty=\sigma_i^{-1}\Gamma_i\sigma_i$ is generated by $\begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$.

We consider $\mathbb{C}$ valued, square integrable functions on $\widehat{\mathbb{H}}$ satisfying $f(\gamma\cdot(z,\phi))=\chi(\gamma)f((z,\phi))$ for any $\gamma\in\Gamma$, and denote the space consists of these funtions as $L^2_\chi(\Gamma,\widehat{\mathbb{H}})$.

Selberg Eigenspace is a subspace of $L^2_\chi(\Gamma,\widehat{\mathbb{H}})$ with two additive conditions of the following;

1. $\dfrac{\partial}{\partial\phi}f=-im\,f$
2. $\Delta f=\lambda f$

We write this space $\mathcal{L}_\chi(m,\lambda)$. Then λ is numbered as follows;

$$\frac{|m|}{2}\left(\frac{|m|}{2}-1\right)\geq\lambda_1\geq\lambda_2\geq\lambda_3\geq\cdots\cdots$$

For the convenience, we set $\lambda_n=-\left(r_n^2+\dfrac{1}{4}\right)=s_n(s_n-1)$, $s_n=\dfrac{1}{2}+ir_n$.

To describe the continuous spectrum, we define the Eisenstein series which attach to κ_i $(i=1,\cdots,\omega)$.

$$E_i(z,\ \phi\ ;s)=\sum_{\{\sigma\}\in\ \Gamma_i\backslash\Gamma} \mathrm{Im}(\ \sigma_i^{-1}\sigma\ z)^s \exp\left(\ -mi\ \sigma_i^{-1}\sigma\ \phi\ \right)\ \chi^{-1}(\sigma)\ P_i$$

$$=\sum \frac{y^s}{|cz+d|^{2s}} \exp\left(-mi\ (\ \phi\ +\ \arg\ (cz\ +d))\right)\ \chi^{-1}(\sigma_i\sigma)\ P_i$$

In the last summation σ runs over all representatives of $\Gamma_\infty\backslash\ \sigma_i^{-1}\Gamma$, and $\sigma=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. P_i is defined as follows;

If κ_i is regular then $P_i=\dfrac{1}{r_i}\displaystyle\sum_{g\in\Gamma_i/\Gamma_i^0}\chi(g)$

If κ_i is irregular then $P_i=\dfrac{1}{2r_i}\displaystyle\sum_{i=1}^{2r_i}(-1)^i\chi(\eta^i)$

where $r_i=[\ \Gamma_i:\ \Gamma_i^0\ ]$, $\eta\in\Gamma_i$ is choosed so that η mod $(\Gamma_i^0)^2$ should be a generater of $\Gamma_i/(\Gamma_i^0)^2$.

This Eisenstein series is meromorphically continued to the whole s-plane and satisfy some functional equation .(cf [2][6]) We write $\Phi_m(s)$ the constant term matrix of these series. When χ is an identity and m is an odd number, $\Phi_m(s)$ is a $\nu\times\nu$ alternative matrix. We can also find that $\Phi_m(\frac{1}{2})$ is unitary if we restrict our attention to the regular cusps. Combining these facts, we know that the number of Γ-inequivalent regular cusps is even .

§2 Selberg Trace Formula for odd weight

First of all we write down Selberg Trace Formula in our case.

Theorem 1 **(Selberg Trace Formula for odd weight)**

Let N be a non negative integer and m=2 N+1. We assume that h(r) is an analytic function in the complex region $|\mathrm{Im}(r)|\leq\max(N,1/2)+\delta$ satisfying following two conditions;

(1) $h(r)=h(-r)$

(2) There exists a sufficiently large number M such as

$$h(r) \leq M * |1+\mathrm{Re}(r)|^{-2-\delta}$$

where δ is some positive real number.

Put $g(u) = \frac{1}{2\pi}\int_{-\infty}^{\infty} h(r)\, e^{-i\pi u}\, du$, then the following formula holds;

$$\sum_{n=1}^{\infty} h(r_n) = \frac{\nu * \mathrm{vol}(\Gamma\backslash\mathbb{H})}{4\pi}\left[\int_{-\infty}^{\infty} r h(r)\coth(\pi r)\, dr + 2\sum_{k=0}^{N} k\, h(ik)\right]$$

$$+ \sum_{\{T\}\in \mathrm{Hyperbolic}} \frac{\mathrm{Tr}(\chi(T))\ \mathrm{sgn}(T)\ \ln N\{T_0\}}{N\{T\}^{\frac{1}{2}} - N\{T\}^{-\frac{1}{2}}}\ g(\ln N\{T\})$$

$$+ \sum_{\{R\}\in \mathrm{Elliptic}} \frac{\mathrm{Tr}(\chi(R))}{4\ {}^{\#}\Gamma(R)\ \sin\theta}\left[\int_{-\infty}^{\infty} h(r)\frac{\sinh(\pi-2\theta)r}{\sinh \pi r}\, dr - i\, h(0) + 2i\sum_{k=0}^{N} e^{2ik\theta}\, h(ik)\right]$$

$$- g(0)\sum_{\substack{\alpha_{ij}\neq 0\\ \text{regular}}} \ln|1-e^{2\pi i\alpha_{ij}}| + \sum_{\substack{\alpha_{ij}\neq 0\\ \text{regular}}}\left(\frac{1}{2}-\alpha_{ij}\right)\left(\sum_{k=0}^{N} h(ik) - \frac{h(0)}{2}\right)$$

$$- g(0)\sum_{\substack{\alpha_{ij}\neq 0\\ \text{irregular}}} \ln|1+e^{2\pi i\alpha_{ij}}| - \sum_{\substack{\alpha_{ij}\neq 0\\ \text{irregular}}} \alpha_{ij}\left(\sum_{k=0}^{N} h(ik) - \frac{h(0)}{2}\right)$$

$$+ \frac{\iota}{4} h(0) - \frac{h(0)}{4}\mathrm{Tr}\left(\Phi_m\left(\frac{1}{2}\right)\right)$$

$$+ \frac{1}{4\pi}\int_{-\infty}^{\infty} h(r)\mathrm{Tr}\left(\Phi'_m\left(\frac{1}{2}+ir\right)\Phi_m\left(\frac{1}{2}+ir\right)^{-1}\right)dr - \iota\, g(0)\ln 2$$

$$+ \frac{\iota}{4\pi}\int_{-\infty}^{\infty} h(r)\left[\psi\left(\frac{1}{2}+ir+\frac{m}{2}\right)+\psi\left(\frac{1}{2}+ir-\frac{m}{2}\right)-2\psi\left(\frac{1}{2}+ir\right)-2\psi(1+ir)\right] dr$$

We regard "~" as the conjugation in $SL(2,\mathbb{R})$ and { } as its conjugacy class . In the above formula we denote that

$T \sim \begin{bmatrix} \lambda & \\ & \lambda^{-1} \end{bmatrix}$, where $|\lambda|>1$, $\Gamma(T)$ means the centralizer of T in Γ, $^{\#}\Gamma(T)$ is the order of this group. T_0 is a generator of $\Gamma(T)$. $N\{T\}= \lambda^2$ $\operatorname{sgn} T= \operatorname{sgn} \lambda$.

$R \sim \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

T_i is a generater of Γ_i, the stabilizer group of cusp κ_i.

$$\chi(T_i) \sim \begin{bmatrix} \exp(2\pi i\alpha_{i1}) & & \\ & \ddots & \\ & & \exp(2\pi i\alpha_{i\nu}) \end{bmatrix}$$

To choose α_{ij} , we determine;

If κ_i is regular then $\alpha_{ij} \in [\ 0\ ,\ 1)$

If κ_i is irregular then $\alpha_{ij} \in [\ -\frac{1}{2}\ ,\ \frac{1}{2}\ )$

T_i can be taken in two ways, as we can replace by T_i^{-1}. So we choose T_i by the clockwise orientation of the normal polygon of $\Gamma\backslash\mathbb{H}$.

ι=A+B where A means the number of pairs (i,j) such that $\alpha_{ij}=0$, where i moves in the range that κ_i is regular and $j=1,\cdots\cdots,\nu$. And B means the number of pairs (i,j) such that $\alpha_{ij}= -\frac{1}{2}$, where i moves in the range that κ_i is irregular and $j=1,\cdots\cdots,\nu$.

$\psi(s)= \frac{\Gamma'}{\Gamma}(s)$ is the digamma function.

§3 Selberg Zeta Function for Odd Weight

Put $Z_\Gamma^*(s,\chi)= \prod_\alpha \prod_{n=0}^{\infty} \det\left(E_\nu - \operatorname{sgn}(P_\alpha)\ \chi(P_\alpha)\ N\{P_\alpha\}^{-s-n} \right)$

where the first $\prod$ runs over all primitive hyperboric conjugacy class $\{P_\alpha\}$ of Γ , and E_ν is the $\nu\times\nu$ unit matrix. Using the formula of §2 under the weaker condition than (2), we can induce the functional equation of $Z_\Gamma^*(s,\chi)$. And in the process we get the *proof* of the formula under the condition (2) .

Now we can write down the functional equation.

Put $$\xi_\Gamma^*(s,\chi)=\frac{d}{ds}\log Z_\Gamma^*(s,\chi)$$

$$=\sum_\alpha\sum_{k=1}^\infty \frac{\mathrm{Tr}(\chi(P_\alpha^k))\ \mathrm{sgn}\ (P_\alpha^k)\ \ln N\{P_\alpha\}}{N\{P_\alpha\}^{\frac{1}{2}}-N\{P_\alpha\}^{-\frac{1}{2}}} N\{P_\alpha\}^{-(s-\frac{1}{2})k}$$

Theorem 2 (**Functional Equation**) We have

$$\xi_\Gamma^*(s,\chi)+\xi_\Gamma^*(1-s,\chi) = -\nu\ \mathrm{vol}(\Gamma\backslash\mathbb{H})\left(s-\frac{1}{2}\right)\cot\left(\pi(s-\frac{1}{2})\right)$$

$$-\pi\sum_{\{R\}}\frac{\mathrm{Tr}(\chi(R))}{{}^{\#}\Gamma(R)\sin\theta}\ \frac{\sin\left((\pi-2\theta)(s-\frac{1}{2})\right)}{\sin\left(\pi(s-\frac{1}{2})\right)}$$

$$+2\sum_{\substack{\alpha_{ij}\neq 0\\ \text{regular}}}\ln|1-e^{2\pi i\alpha_{ij}}| + 2\sum_{\substack{\alpha_{ij}\neq 0\\ \text{irregular}}}\ln|1+e^{2\pi i\alpha_{ij}}|$$

$$-\frac{\varphi'}{\varphi}(s)-\frac{\iota}{2}\left(\xi_m(s)+\xi_m(1-s)\right)+2\iota\ln 2$$

where $\xi_m(s)=\psi(s+\frac{m}{2})+\psi(s-\frac{m}{2})-2\psi(s)-2\psi(s+\frac{1}{2})$,

$\varphi(s)=\det\Phi_m(s)$

Especially in the case when Γ contains no elliptic elements and has compact fundamental region, $Z_\Gamma^*(s,\chi)$ has a simillar functional equation with the classical Selberg Zeta function.

Cor 2.1

Suppose Γ is strictly hyperbolic then

$$Z_\Gamma^*(s,\chi)=Z_\Gamma^*(1-s,\chi)*\exp\left(-\nu*\mathrm{vol}(\Gamma\backslash\mathbb{H})\int_0^{s-\frac{1}{2}} z\cot(\pi z)\,dz\right)$$

§4 Duality of Riemann-Roch type and the dimension of the space of cusp forms of weight one.

First we consider the space $\mathcal{S}_m(\Gamma,\chi)$ consists of ν row vector valued holomorphic functions satisfying 1. and 2.

1. $F|[T]_m = \chi(T)\ F \qquad \text{for } T \in \Gamma$

2. $\displaystyle\int_{\Gamma\backslash\mathbb{H}} F(z)\ {}^t\overline{F(z)}\ y^m\ dz < \infty$

where $F|[T]_m = F(T\cdot z)\ j(T,z)^{-m}$. The connextion of this space $\mathcal{S}_m(\Gamma,\chi)$ and Selberg Eigenspace is given by the following lemma.

Lemma 1.

$$\mathcal{L}_\chi\left(m+2, \frac{m}{2}\left(1+\frac{m}{2}\right)\right) = y^{\frac{m+2}{2}} \exp\left(-i(m+2)\phi\right)\ \mathcal{S}_{m+2}(\Gamma,\chi)$$

$$\mathcal{L}_\chi\left(m, \frac{m}{2}\left(1+\frac{m}{2}\right)\right) = y^{-\frac{m}{2}} \exp\left(-im\ \phi\right)\ \overline{\mathcal{S}_{-m}(\Gamma,\overline{\chi})}$$

Lemma 2.

Suppose $\lambda \neq \frac{m}{2}\left(1+\frac{m}{2}\right)$ then $\qquad \dim \mathcal{L}_\chi(m,\lambda) = \dim \mathcal{L}_\chi(m+2,\lambda)$.

Using these two lemmas, we can caluculate the difference of the dimension of $\mathcal{S}_m(\Gamma,\chi)$ between that of $\mathcal{S}_{2-m}(\Gamma,\chi)$.

Theorem 3

$\dim \mathcal{S}_m(\Gamma,\chi) - \dim \mathcal{S}_{2-m}(\Gamma,\overline{\chi})$

$$= \frac{\nu\ \mathrm{vol}(\Gamma\backslash\mathbb{H})}{4\pi}\cdot(m-1) + \sum_{\{R\}} \frac{\mathrm{Tr}(\chi(R))\, i\ e^{i(m-1)\theta}}{2\ {}^{\#}\Gamma(R)\ \sin\theta}$$

$$+ \sum_{\substack{\alpha_{ij}\neq 0 \\ \text{regular}}} \left(\frac{1}{2} - \alpha_{ij}\right) - \sum_{\substack{\alpha_{ij}\neq 0 \\ \text{irregular}}} \alpha_{ij}$$

$$- \quad \frac{\iota}{2}\,\mathrm{sgn}(m-1) \quad - \frac{\delta_{m1}}{2}\,\mathrm{Tr}\left(\Phi_m(\tfrac{1}{2})\right)$$

where $\mathrm{sgn}(0)=0$ and δ_{ij} is the Kronecker symbol.

If $m \geq 1$ then the space $\mathcal{S}_m(\Gamma,\chi)$ coincides with $S_m(\Gamma,\chi)$, the space of classical cusp forms (of ν row vector valued). And $m<0$ then $\mathcal{S}_m(\Gamma,\chi)$ is empty. Although $S_0(\Gamma,\chi)$ is empty , $\mathcal{S}_0(\Gamma,\chi)$ is the space of constant functions of dimension one. Seeing these facts, we can derive the dimension formula of $S_m(\Gamma,\chi)$ for m:odd and $\Gamma \not\ni -1$.

<u>Cor 3.1</u>

For $m \geq 3$, $m=2N+1$

$d_m = \dim S_m(\Gamma,\chi)$

$$= \frac{\nu\,\mathrm{vol}(\Gamma\backslash\mathbb{H})}{2\pi}\,N + i \sum_{\{R\}} \frac{\mathrm{Tr}(\chi(R))\,e^{2N\theta}}{2\,{}^{\#}\Gamma(R)\,\sin\theta}$$

$$+ \sum_{\substack{\alpha_{ij}\neq 0\\ \text{regular}}} \left(\frac{1}{2} - \alpha_{ij}\right) - \sum_{\substack{\alpha_{ij}\neq 0\\ \text{irregular}}} \alpha_{ij} \quad - \frac{\iota}{2}$$

In the case of weight one, we can only catch the difference between $S_m(\Gamma,\chi)$ and $S_m(\Gamma,\overline{\chi})$.

<u>Cor 3.2</u>

$\dim S_1(\Gamma,\chi) - \dim S_1(\Gamma,\overline{\chi})$

$$= \quad i \sum_{\{R\}} \frac{\mathrm{Tr}(\chi(R))}{2\,{}^{\#}\Gamma(R)\,\sin\theta}$$

$$+ \sum_{\substack{\alpha_{ij}\neq 0\\ \text{regular}}} \left(\frac{1}{2} - \alpha_{ij}\right) - \sum_{\substack{\alpha_{ij}\neq 0\\ \text{irregular}}} \alpha_{ij} \quad - \frac{1}{2}\mathrm{Tr}\left(\Phi_1(\tfrac{1}{2})\right)$$

§5 The other expression of d_1

Now we treat the Trace Formula of §2 as a one parameter family. We consider that $h(r)$ is not only a function of r but also of s, where s runs over all complex plane and $h(r)=h(r,s)$ is a meromorphic function of r and s. We assume that the Trace Formula converges in some *region* of s, then we may use the Trace Formula for the *larger region* if the Trace Formula is analytically continued.

As $d_m=\dim S_m(\Gamma,\chi)$ is the multiplicity of $h(\frac{1-m}{2}i)$ in the left hand side of the Trace Formula, we can catch the value d_m if $h(\frac{1-m}{2}i,s)$ has a pole at some $s=s_1$ and $h(r,s)$ is holomorphic at $s=s_1$ for another r_n. According to this criterion, we can again induce Cor 3.1. In this case the hyperbolic term corresponds to the holomorphic point of $\xi^*_\Gamma(s,\chi)$, so the contribution to the dimension formula vanishes. Using the criterion for d_1, we can induce Theorem 4 . We notice that this is somewhat different form from Cor 3.2.

Theorem 4 We have

$$d_1 = \frac{1}{2}\ \underset{s=\frac{1}{2}}{\mathrm{ord}_1}\ Z^*_\Gamma(s,\chi) + i \sum_{\{R\}} \frac{\mathrm{Tr}(\chi(R))}{4\ {}^{\#}\Gamma(R)\sin\theta}$$

$$+ \frac{1}{2} \sum_{\substack{\alpha_{ij}\neq 0\\ \text{regular}}} \left(\frac{1}{2} - \alpha_{ij}\right) - \frac{1}{2} \sum_{\substack{\alpha_{ij}\neq 0\\ \text{irregular}}} \alpha_{ij} - \frac{1}{4}\mathrm{Tr}\left(\Phi_1\left(\frac{1}{2}\right)\right)$$

where "ord" means the order of zeros.

Comparing with Cor 3.2 we get

Theorem 4'

$$\dim S_1(\Gamma,\chi) + \dim S_1(\Gamma,\overline{\chi}) = \underset{s=\frac{1}{2}}{\mathrm{ord}}\ Z^*_\Gamma(s,\chi)$$

Cor 4.1

For real representation χ , $\quad d_1 = \frac{1}{2} \operatorname{ord}_{s=\frac{1}{2}} Z^*_\Gamma(s,\chi)$

This result is the good explanation of [2][6][7].
Comparing Trace Formulas of different odd weight, we easily get the following.

Theorem 5

$$d_1=\dim \mathcal{L}_\chi(2N+1,-\frac{1}{4}) + \frac{1}{2}\mathrm{Tr}\ \Phi_1\left(\frac{1}{2}\right) \qquad \text{for } N \geq 1$$

From Lemma 2 we know that $\dim \mathcal{L}_\chi(2N+1,-\frac{1}{4})$ is independent of N.
This result can be written in the form

$$\dim \mathcal{L}_\chi(1,-\frac{1}{4}) - \dim \mathcal{L}_\chi(3,-\frac{1}{4}) = \frac{1}{2}\mathrm{Tr}\ \Phi_1\left(\frac{1}{2}\right)$$

If χ is an identity then Φ_1 is alternative so that the right hand side is zero.

Cor 5.1

Let χ be an identity , $\quad d_1=\dim \mathcal{L}_\chi(2N+1,-\frac{1}{4}) \quad$ for $N \geq 0$

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References

[1] Hejhal.D.A: The Selberg Trace Formula for PSL(2,ℝ) vol 1,2 , Lecture Notes in Math., vol **548** & **1001**, Springer, Berlin (1976, 1983)

[2] H.Ishikawa and Y.Tanigawa: The dimension formula of the space of

cusp forms of weight one for $\Gamma_0(p)$, Proc Japan Acad **63**, ser.A, no.2, (1987) 31-34

[3] Selberg.A: Harmonic analysis and discontinuous groups in weakly symmetric Riemannian space with application to Dirichlet series, J.Indian Math. Soc. **20** ,47-87 (1956)

[4] Selberg.A: Discontinuous groups and harmonic analysis, Proc. International Congr. Math. 177-189 (1962)

[5] S.Akiyama, T.Hiramatsu and Y.Tanigawa: On the traces of Hecke operaters on the space of cusp forms of weight one, preprint

[6] S.Akiyama and T.Hiramatsu: On some dimension formula for automorphic forms of weight one Ⅲ , Nagoya Math. J. to apear.

[7] T.Hiramatsu: On some dimension formula for automorphic forms of weight one Ⅰ,Ⅱ , Nagoya Math.J. **85**, 213-221 (1982) & Nagoya Math.J. **105**, 169-186 (1987)

[8] T.Kubota: Elementary theory of Eisenstein series, Tokyo-New York, Kodansha and Halsted (1973)